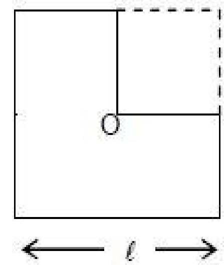
ALTUS KNOWLEDGE

Total: 75 Que.	IIT Full Length 3	Time: 180 Min.

Physics FL

1).

One quarter of the plate is cut from a square plate as shown in the figure. If 'M' is the mass of the plate and 'ℓ' is the length of each side, then the moment of inertia of the plate about an axis passing through 'O' and perpendicular to the plate is



(A) $M\ell^2 / 8$ (B) $3M\ell^2/4$

(C) $M\ell^2/3$ (D) $3M\ell^2$

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

For full square about an axis passing through 'O' $= \dfrac{M\ell^2}{6}$

by symmetry for remaining portion it must be $\dfrac{3}{4}\left[\dfrac{M\ell^2}{6}\right] = \dfrac{M\ell^2}{8}$

2).

A block is suspended by an ideal spring constant K. If the block is pulled down by constant force F and if maximum displacement of block from it's initial position of rest is z, then

(A) $z = F/K$ (B) $z = 2F/K$

(C) work done by force F is equal to 2Fz.

(D) increase in potential energy of the spring is $\dfrac{1}{2}Kz^2$

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

apply work-energy theorem

$$mgz - \left[\frac{1}{2}K\left(\frac{mg}{K}+z\right)^2 - \frac{1}{2}K\left(\frac{mg}{K}\right)^2\right] + Fz = 0$$

$\Rightarrow z = 2F/K.$

3).

A Carnot engine whose low temperature reservoir is at 7°C has an efficiency of 50%. It is desired to increase the efficiency to 70%. By how many degrees should the temperature of the high temperature reservoir be increased

(a) 840K (b) 280 K (c) 560 K (d) 380K

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

$$\eta_A = \frac{T_1 - T_2}{T_1} = \frac{W_A}{Q_1} \Rightarrow \eta_B = \frac{T_2 - T_3}{T_2} = \frac{W_B}{Q_2}$$

$$\therefore \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \times \frac{T_2 - T_3}{T_1 - T_2} = \frac{T_1}{T_2} \therefore W_A = W_B$$

$$\therefore T_2 = \frac{T_1 + T_3}{2} = \frac{800 + 300}{2} = 550K$$

4).

A boat goes downstream for half an hour and then goes upstream for half an hour. The total distance travelled by the boat in the ground frame for this is 20 km. Ii is known that speed of the boat relative to the river for the whole trip was constant and greater than the speed of the river. The distance travelled by the boat in the frame of the river for this is

(A) zero (B) 20 km
(C) 10 km (D) can't be determined

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

Say speed of boat is v w.r.t. water and speed of river is C. Then, distance travelled in ground frame

$= (c + v) \times \frac{1}{2}$ hour $+ (v - c) \times \frac{1}{2}$ hour $= v \times 1$ hour

= distance travelled by boat w.r.t. river.

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5).

For a certain organ pipe open at both ends, the successive resonance frequencies are obtained at 510, 680 and 850 Hz. The velocity of sound in air is 340 m/s. The length of the pipe must be

(A) 2 m (B) 0.5 m

(C) m (D) 0.25 m

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

Successive frequencies will differ by an amount v/2L

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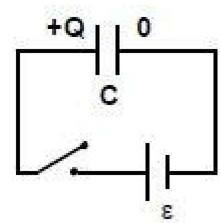
6).

The left plate of the capacitor shown in the figure above carries a charge +Q while the right plate is uncharged at t = 0. The total charge on the right plate after closing the switch will be



(A) $\frac{Q}{2} + C\varepsilon$ (B) $\frac{Q}{2} - C\varepsilon$

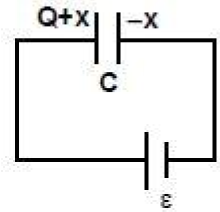
(C) $-\frac{Q}{2}$ (D) $- C\varepsilon$

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

Electric field between the capacitor plates = $\frac{\sigma_1}{2\varepsilon_0} + \frac{(-\sigma_2)}{2\varepsilon_0}$



$$E = \frac{Q+x}{2A\varepsilon_0} + \frac{x}{2A\varepsilon_0} = \frac{1}{2A\varepsilon_0}[Q+2x]$$

$\Rightarrow$ Potential different $E_d = \frac{d}{2A\varepsilon_0}[Q+2x] = \varepsilon$

$$\Rightarrow \varepsilon = \frac{Q+2x}{2C}$$

$$\Rightarrow -x = \frac{Q}{2} - C\varepsilon$$

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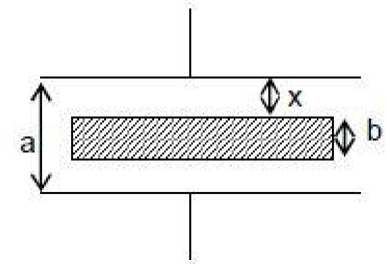
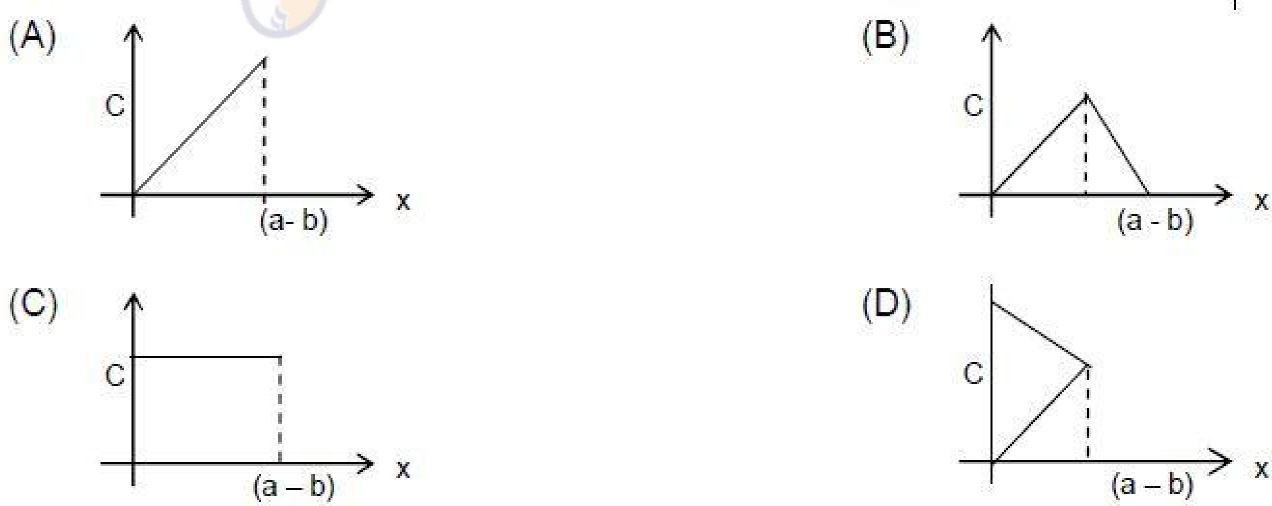
7).

The distance between two parallel plates of a capacitor is a. A conductor of thickness b(b < a) is inserted between the plates as shown in the figure. The variation of effective capacitance between the surfaces of conductor and plate as a function of the distance (x) is best represented by





(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{x}{\varepsilon_0 A} + \frac{a-b-x}{\varepsilon_0 A} \Rightarrow C = \frac{\varepsilon_0 A}{a-b}$$

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8).

A block of mass 2 kg is attached to one end of a massless rod of length $\frac{1}{\pi}$ m. The rod is fixed to a horizontal plane at the other end such that the block and rod are free to revolve on a horizontal plane. The coefficient of friction between the block and surface is 0.1. Block is made to rotate with uniform speed by applying a constant external force in tangential direction on the block. The work done by external force when the rod rotates by 90^0 is

(A) 0
(B) 10 joule
(C) $\frac{\pi}{2}$ joule
(D) 1 joule

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** D

Solution :-

$$W = \int_0^{\pi/2} f.Rd\theta = \frac{\mu mgR\pi}{2} = 1 \text{ joule.}$$

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9).

A solid sphere of radius R, and dielectric constant 'k' has spherical cavity of radius R/4. A point charge q_1 is placed in the cavity. Another charge q_2 is placed outside the sphere at a distance of r from q. Then Coulombic force of interaction between them is found to be 'F_1'. When the same charges are separated by same distance in vacuum then the force of interaction between them is found to be F_2 then

(A) $F_1 = F_2/k$
(B) $F_2 = F_1/k$
(C) $F_1 . F_2 = \frac{1}{k}$
(D) $F_1 = F_2$

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** D

Solution :-

Coulombic force between them remains same.

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10).

A swimmer can swim in still water with a speed of $\sqrt{5}$ m/s. While crossing a river his average speed is 3 m/s. If he cross the river in the shortest possible time, what is the speed of flow of water?

(A) 2 m/s
(B) 4 m/s
(C) 6 m/s
(D) 8 m/s

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

$$\text{Avg. speed } 3 = \frac{\sqrt{(v_r t)^2 + (v_{mr} t)^2}}{t}$$
$$\Rightarrow v_r^2 + 5 = 9$$
$$\Rightarrow v_r = 2 m/s$$

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11).

In the hydrogen atom spectrum λ_{3-1} and λ_{2-1} represent wavelengths emitted due to transition from second and first excited states to the ground state respectively. The value of $\frac{\lambda_{3-1}}{\lambda_{2-1}}$ is

(A) 27 / 32 (B) 32/27
(C) 4/9 (D) 9/4

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

$$\frac{1}{\lambda_{3-1}} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8R}{9}$$
$$\frac{1}{\lambda_{2-1}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$
$$\Rightarrow \frac{\lambda_{3-1}}{\lambda_{2-1}} = \frac{27}{32}$$

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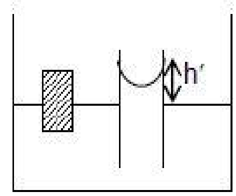
12).

In a capillary tube placed inside the liquid of density (ρ)in a container, the rise of liquid is h. When block of density 'σ' is placed on the liquid as shown in figure, liquid in the tube is h′. If $\sigma < \rho$ then



(A) h′ = h (B) h′ < h
(C) h′ > h (D) insufficient data

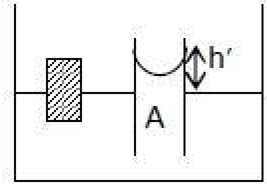
(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

There will be no change.

$\therefore \quad h' = h$



13).

The power factor of a circuit in which a box having unknown electrical devices connected in series with a resistor of resistance 3Ω is 3/5. The reactance of the box is

(A) 5 Ω (B) 5/3 Ω

(C) 4 Ω (D) 4/3 Ω

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

$Z = \sqrt{R^2 + X^2} = \sqrt{9 + X^2}$

but $\cos\phi = \frac{R}{Z} = \frac{3}{5}$

$X = 4\ \Omega.$

14).

Two points A and B are at distances of 'a' and 'b' respectively from an infinite conducting plate having charge density σ. The work done in moving charge Q from A to B is

(A) $\frac{Q\sigma}{\varepsilon_0}(b-a)$ (B) $\frac{\sigma}{(b-a)}Q$

(C) $\frac{Q\sigma}{(b-a)\varepsilon_0}$ (D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

$$\varepsilon = \frac{\sigma}{\varepsilon_0}$$

$$v_A - v_B = \frac{\sigma}{\varepsilon_0}(b) - \frac{\sigma}{\varepsilon_0}(a)$$

$$\therefore\ W = Q(v_A - v_B) = \frac{Q\sigma}{\varepsilon_0}(b - a)$$

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15).

A point charge of 0.1C is placed on the circumference of a non-conducting ring of radius 1m which is rotating with a constant angular acceleration of 1 rad/sec^2. If ring starts it's motion at t = 0 the magnetic field at the centre of the ring at t = 10 sec, is

(A) 10^{-6} T (B) 10^{-7} T
(C) 10^{-8} T (D) 10^{7} T

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** D

Solution :-

$$\omega = 0 + 1 \times 10 = 10 \text{ rad/sec}^2$$

$$\therefore\ v = r\omega = 1 \times 10 = 10 \text{ m/s}$$

$$\bar{B} = \frac{\mu_0}{4\pi}\frac{q(\bar{v} \times \bar{r})}{r^3} \Rightarrow |\bar{B}| = \frac{\mu_0 q v}{4\pi r^2}$$

$$B = \frac{10^{-7} \times 0.1 \times 10}{(1)^2} = 10^{-7}\text{ T}$$

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16).

The wavelength corresponding to maximum spectral radiancy of a black body A is λ_A = 5000 Å. Consider another black body B whose surface area is twice of that of A and total radiant energy emitted by B is 16 times that emitted by A. The wavelength corresponding to maximum spectrum radiancy for B will be

(A) $5000\,(8)^{1/4}$ Å (B) 2500 Å
(C) 10, 000 Å (D) $\frac{5000}{(8)^{1/4}}$ Å

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** D

Solution :-

$P = \sigma A T^4$

$$\Rightarrow \frac{P_A}{P_B} = \frac{A_B}{A_A}\left(\frac{T_B}{T_A}\right)^4 \Rightarrow 16 = \frac{2}{1}\left(\frac{T_B}{T_A}\right)^4 \Rightarrow T_B = T_A(8)^{1/4}$$

Since $\lambda_m T$ = constant, $\frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A} = (8)^{1/4}$

$$\Rightarrow \lambda_B = \frac{\lambda_A}{(8)^{1/4}} = \frac{5000}{(8)^{1/4}} \text{ Å}$$

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17).

Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube A and B is

(a) 1 : 2 (b) 1 : 4
(c) 2 : 1 (d) 4 : 1.

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

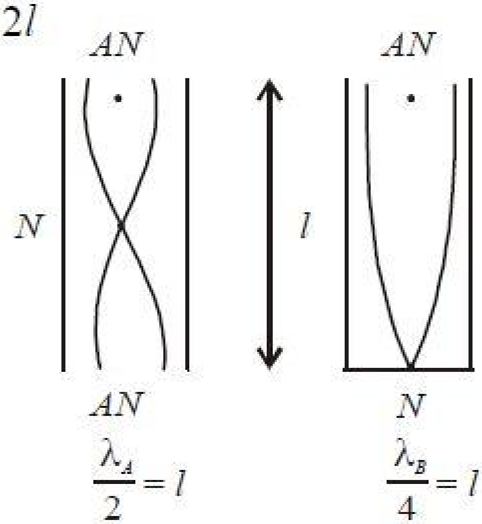
Solution :-

(c) : In tube A, $\lambda_A = 2l$
In tube B, $\lambda_B = 4l$

$$\therefore \upsilon_A = \frac{v}{\lambda_A} = \frac{v}{2l}$$

$$\upsilon_B = \frac{v}{\lambda_B} = \frac{v}{4l}$$

$$\therefore \frac{\upsilon_A}{\upsilon_B} = \frac{2}{1}.$$



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18).

The angle of incidence at which reflected light in totally polarized for reflection from air to glass (refractive index n), is

(a) $\sin^{-1}(n)$ (b) $\sin^{-1}(1/n)$
(c) $\tan^{-1}(1/n)$ (d) $\tan^{-1}(n)$.

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** D

Solution :-

(d) : According to Brewster's law of polarization,

$n = \tan i_p$ where i_p is angle of incidence

$\therefore \quad i_p = \tan^{-1}(n)$.

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19).

What is the maximum acceleration of the particle doing the SHM $y = 2\sin\left[\frac{\pi t}{2} + \phi\right]$ where y is in cm?

(a) $\frac{\pi}{2}$ cm/s^2 (b) $\frac{\pi^2}{2}$ cm/s^2

(c) $\frac{\pi}{4}$ cm/s^2 (d) $\frac{\pi}{4}$ cm/s^2

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

Comparing given equation with standard equation,

$y = a\sin(\omega t + \phi)$, we get, $a = 2cm, \omega = \frac{\pi}{2}$

$\therefore A_{\max} = \omega^2 A = \left(\frac{\pi}{2}\right)^2 \times 2 = \frac{\pi^2}{2} cm/s^2$

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20). If I_0 is the intensity of the principal maximum in the single slit diffraction pattern, then what will be its intensity when the slit width is doubled?

(a) I_0 (b) $I_0/2$ (c) $2I_0$ (d) $4I_0$

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

(a) : For diffraction pattern

$I = I_0\left(\frac{\sin\phi}{\phi}\right)^2$ where ϕ denotes path difference

For principal maxima, $\phi = 0$. Hence $\left(\frac{\sin\phi}{\phi}\right) = 1$

Hence intensity remains constant at I_0

$I = I_0\ (1) = I_0$.

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21).

A particle starts from rest and moves with an acceleration of a = {2 + |t – 2|} m/s^2, the velocity of the particle at t = 4 sec is

Q.Type:- Digit/Numeric, **Ans:-** 12

Solution :-

a = 2 + |t – 2|

for t ≤ 2

a = 2 – t + 2

a = 4 – t

dv = (4 – t)dt

v = 4t – t^2/2

at t = 2, v = 6 m/s.

for t > 2

a = 2 + t – 2 = t

$\int_6^v dv = \int_2^t t\,dv$

$v - 6 = \left[t^2/2\right]_2^t$

$v = \frac{t^2}{2} + 4$

at t = 4, v = 12 m/s.

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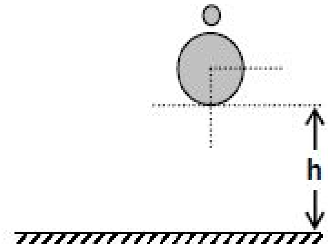
22).

A small sphere and a big sphere are released from rest with a very small gap from height h as shown in the figure. The mass of bigger sphere is very large as compared to mass of smaller sphere the height from the point of collision of smaller sphere with the bigger sphere to which the smaller sphere will rise if all the collisions are elastic



Q.Type:- Digit/Numeric, **Ans:-** 9

Solution :-

conservation of momentum

$$M\sqrt{2gh} - m\sqrt{2gh} = MV_1 + mV_2 \quad (1)$$

$$-1 = \frac{V_1 - V_2}{2\sqrt{2gh}} \quad (2)$$

$$V_2 = 3\sqrt{2gh}$$

$$h' = \frac{V_2^2}{2g} = 9h$$

23). At what height above the earth's surface the acceleration due to gravity will be 1/9 th of its value at the earth's surface? Radius of earth is 6400 km.

Q.Type:- Digit/Numeric, **Ans:-** 12800

Solution :-

If g be the acceleration due to gravity at the surface of the earth, then its value at a height h above the earth's surface will be -

$$g' = \frac{g}{\left(1+\frac{h}{R_e}\right)^2} \qquad \text{Here} \quad \frac{g'}{g} = \frac{1}{9}$$

$$\therefore \frac{1}{9} = \frac{1}{\left(1+\frac{h}{R_e}\right)^2} \text{ or } 1 + \frac{h}{R_e} = 3$$

$$\text{or } h = 2R_e = 2 \times 6400 = 12800 \text{ km.}$$

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24).

During an adiabatic process, the density of a gas is found to be proportional to cube of temperature. The degree of freedom of gas molecule is

Q.Type:- Digit/Numeric, **Ans:-** 6

Solution :-

For adiabatic process, $TV^{\gamma-1}$ = constant

$$T\left(\frac{m}{\rho}\right)^{\gamma-1} = \text{constant}$$

$$\frac{T}{\rho^{\gamma-1}} = \text{constant}$$

$$\rho \propto T^{1/(\gamma-1)} \Rightarrow \frac{1}{\gamma-1} = 3 \Rightarrow \gamma = 4/3$$

$$f = \frac{2}{\gamma-1} = \frac{2}{\left(\frac{4}{3}-1\right)} = 6$$

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25).

An alternating voltage $E = 200\sqrt{2}\sin(100t)$ is connected to a 1 microfarad capacitor through an ac ammeter. The reading of the ammeter will be

Q.Type:- Digit/Numeric, **Ans:-** 20

Solution :-

Reading of ammeter,

$$i_{rms} = \frac{V_{rms}}{X_c} = \frac{V_o \omega C}{\sqrt{2}}$$

$$= \frac{200\sqrt{2} \times 100 \times (1 \times 10^{-6})}{\sqrt{2}}$$

$$= 2 \times 10^{-2}\ \text{A} = 20\ \text{m A}$$

Chemistry FL

26).

A compound is composed of two elements A and B, element A constitute f.c.c. lattice, while B occupy all the tetrahedral voids, in this way another simple cubic is constituted by element B, inside the fcc unit cell of element A. If all the points/particles along any one edge of inner cube of every unit cell, are missing then what is the new empirical formula of the compound

(A) A_4B_6 (B) A_2B_8

(C) A_2B_3 (D) AB_4

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

2B atoms (ions are missing from every unit cell).

27).

The ratio of time periods taken by electron in 1st and 3rd orbits of He^{+} ion, for each revolution is.....

(A) 1 : 9 (B) 1 : 27

(C) 8 : 27 (D) 8 : 9

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

$$\text{Time period} \propto \frac{n^3}{z^3}$$

$$\frac{T_1}{T_2} = \frac{n_1^3 \times z_2^3}{n_2^3 \times z_1^3};\ z_1 = z_2 \text{ for same element.}$$

$$\text{Hence, } \frac{T_1}{T_2} = \frac{1^3}{3^3} = \frac{1}{27}$$

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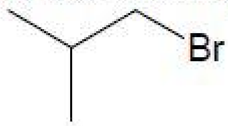
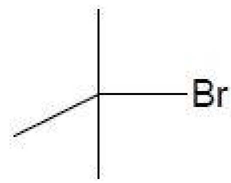
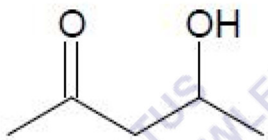
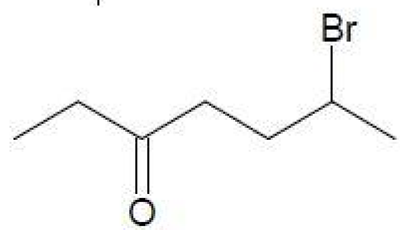
28). Which of the following will undergo $E1_{cB}$.?

(A) 

(B) 

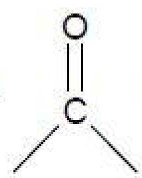
(C) 

(D) 

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

Leaving group is at β position to anion stabilizing group (i.e.  group.)

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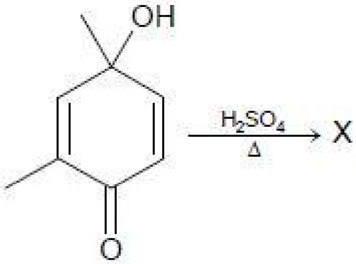
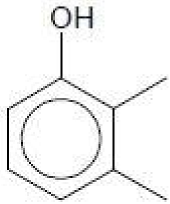
29).

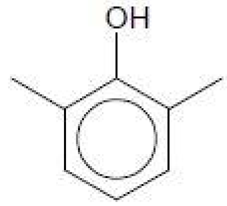


Product X is :

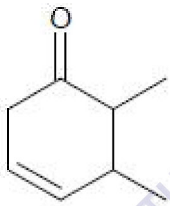
(A)



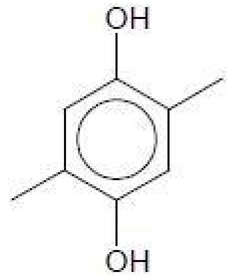
(B)



(C)



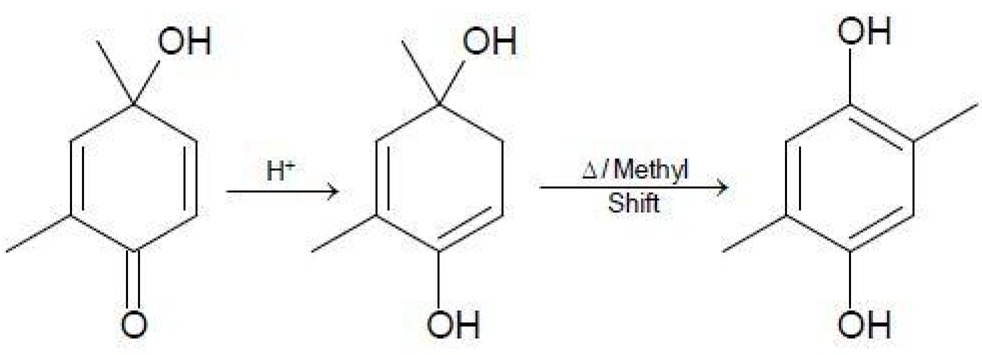
(D)



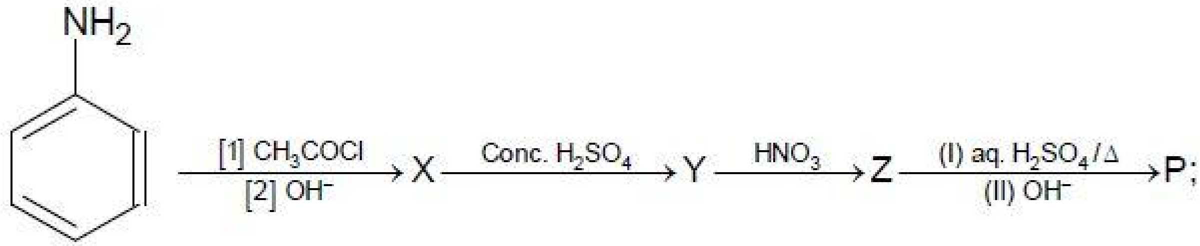
(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** D

Solution :-

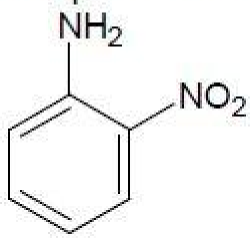


30).

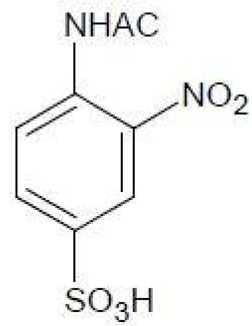


What is the product 'P' of the above reaction

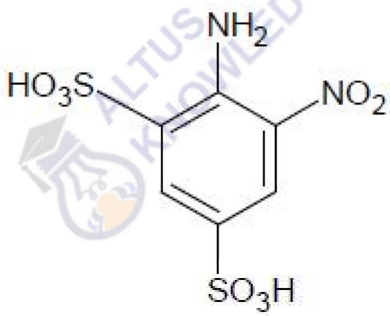
(A)



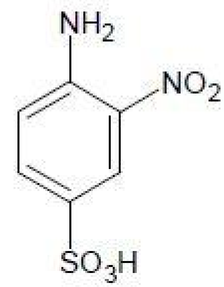
(B)



(C)



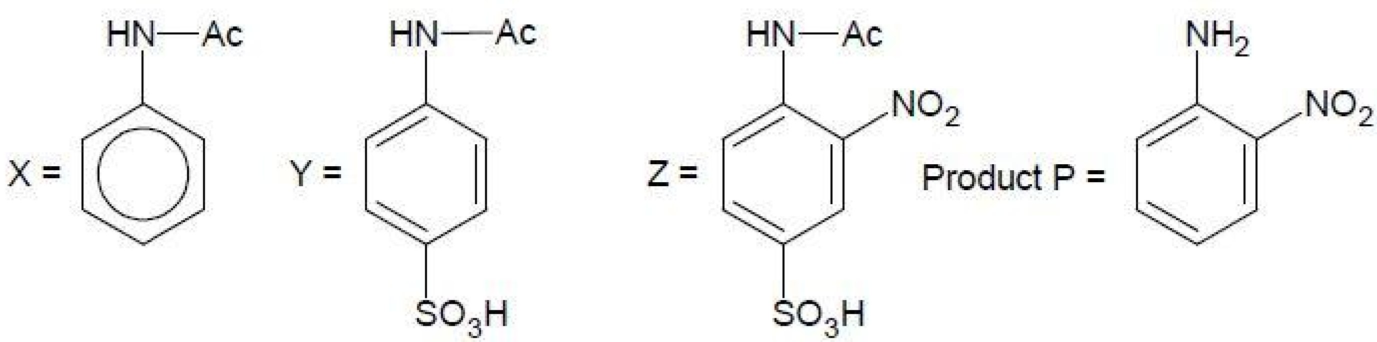
(D)



(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-



31). Which of the following is tetrahedral and paramagnetic complex?

(A) $[NiCl_4]^{2-}$ (B) $[Ni(CN)_4]^{2-}$

(C) $[Cu(NH_3)_4]^{2+}$ (D) $[Ni(CO)_4]$

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

$[NiCl_4]^{2-} \Leftarrow dsp^2$, square planer and diamagnetic

$[Cu(NH_3)_4]^{2+} \Leftarrow dsp^2$, square planer and paramagnetic

$[Ni(CO)_4] \Leftarrow$ is sp^3, tetrahedral but diamagnetic

---

32).

What is the solubility product of CaF_2 at room temperature, if $\overset{o}{\Lambda}_m(Ca^{2+}) = 1.04 \times 10^{-2}$ Sm^2/mol

$\overset{o}{\Lambda}_m(F^-) = 4.8 \times 10^{-3}$ Sm^2/mol

$\kappa_{CaF_2(\text{Saturated Solution})} = 4.25 \times 10^{-3}$ S / m at room temperature.

$\kappa_{H_2O} = 2 \times 10^{-4}$ S / m

(A) 4.05×10^{-4} M^3
(B) 2.025×10^{-4} M^3
(C) 3.32×10^{-11} M^3
(D) 4.05×10^{10} M^3

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

$\kappa_{CaF_2} = \kappa_{CaF_2}$ (solution) - κ_{H_2O}

$= 4.05 \times 10^{-3}$ S/m

$\text{Conc} = \dfrac{\kappa}{\Lambda_m} = 2.025 \times 10^{-4}$ mol / dm^3

$K_{sp} = 4C^3 = 3.32 \times 10^{-11} M^3$

---

33). By which of the following methods, Cl_2 can be produced?

(A) By treating $KClO_3$ with iodine (I_2)
(B) By heating a mixture of NaBr and $NaBrO_3$ with HCl
(C) By treating $CaOCl_2$ and NaI mixture with HCl solution.
(D) By treating NaCl with H_2SO_4

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

(A) $2KClO_3 + I_2 \longrightarrow 2KIO_3 + Cl_2$

(B) $5NaBr + NaBrO_3 + 6HCl \longrightarrow 6NaCl + 3Br_2 + 3H_2O$

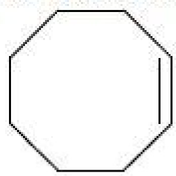
(C) $CaOCl_2 + 2NaI + 2HCl \longrightarrow I_2 + CaCl_2 + H_2O + NaCl$

(D) $NaCl + H_2SO_4 \longrightarrow NaHSO_4 + HCl$

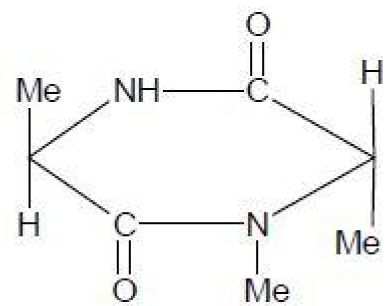
---

34). Which of the following is optically active?

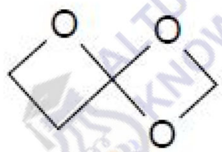
(A)



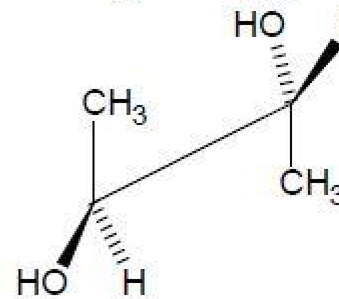
(B)



(C)



(D)



(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

A and C possesses plane of symmetry. While D possesses centre of symmetry.

---

35).

The rate of effusion of two gases 'a' and 'b' under identical condition of temperature and pressure are in the ratio of 2 : 1. What is the ratio of rms velocity of their molecules if T_a and T_b are in the ratio of 2 : 1?

(A) 2 : 1 (B) $\sqrt{2}:1$

(C) $2\sqrt{2}:1$ (D) $1:\sqrt{2}$

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

$$\frac{r_a}{r_b} = \frac{2}{1} = \sqrt{\frac{M_b}{M_a}}$$

$$V_{rms} \propto \sqrt{\frac{T}{M}} \qquad \left(\text{As } V_{rms} = \sqrt{\frac{3RT}{M}}\right)$$

$$\frac{V_{rms(a)}}{V_{rms(b)}} = \sqrt{\frac{T_a \times M_b}{T_b \times M_a}} = \frac{2}{1} \times \frac{\sqrt{2}}{1} = \frac{2\sqrt{2}}{1}$$

---

36).

A complex of iron and cyanide ions is 100% ionized at 1 molal. If its elevation in boiling point is 2.08^{o}, then the complex is: (Given $K_b = 0.52^{o}C\ mol^{-1}\ kg$)

(A) $Fe_4[Fe(CN)_6]_3$ (B) $K_3[Fe(CN)_6]$

(C) $K_4[Fe(CN)_6]$ (D) $Fe_3[Fe(CN)_6]_2$

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

$\Delta T_b = i \times k_b \times m$

$2.08 = 0.52 \times 1 \times i$

$i = 4$

It means salt on dissociation gives 4 ions. Thus the salt that gives 4 ions is $K_3[Fe(CN)_6]$.

---

37).

Find the standard cell potential involving the cell reaction:

$In^{2+} + Cu^{+2} \longrightarrow In^{+3} + Cu^{+}$, at 298 K

Given: $E^{0}_{Cu^{+2}/Cu^{+}} = X_1V$, $E^{0}_{In^{+3}/In^{+}} = X_2V$ $E^{o}_{In^{+2}/In^{+}} = X_3V$

(A) $X_1 + X_3 - X_2$ (B) $(X_1 + X_3 - 2X_2)/3$

(C) $X_1 + X_3 - 2X_2$ (D) $X_1 + X_3 + 2X_2$

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

$$Cu^{+2} + e^- \longrightarrow Cu^+; \quad E^o = X_1 V \quad \Delta G_1^o = -FX_1 \quad \text{... (i)}$$

$$In^{+3} + 2e^- \longrightarrow In^{+1} \quad E^o = X_2 V \quad \Delta G_2^o = -2FX_2 \quad \text{...(ii)}$$

$$In^{+2} + e^- \longrightarrow In^+ \quad E^o = X_3 V \quad \Delta G_3^o = -FX_3 \quad \text{... (iii)}$$

From Eqn. (i) + (iii) – (ii)

$$In^{+2} + Cu^{2+} \longrightarrow In^{+3} + Cu^+ \qquad \Delta G^o = -FE^o$$

$$\Delta G^o = -F(X_1 + X_3 - 2X_2) = -FE^o$$

$$E^o = (X_1 + X_3 - 2X_2)$$

---

38).

An original salt solution in acidic medium did not give any precipitate on passing H_2S gas. Such a solution was boiled, reboiled after dilution 3 times. To such a solutions two drops of conc. HNO_3 were added, then heated and water was added. To this resulting solution, NH_4Cl was first added followed by excess of NH_4OH. Finally a green ppt. was obtained. Hence the cation may be:

(A) Al^{+3} (B) Fe^{+2}
(C) Fe^{+3} (D) Cr^{+3}

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** D

Solution :-

Green ppt. is – $Cr(OH)_3$

---

39).

$$H_3CCOOH \xrightarrow{Br_2/P} (Y) \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) KCN}} (X)$$

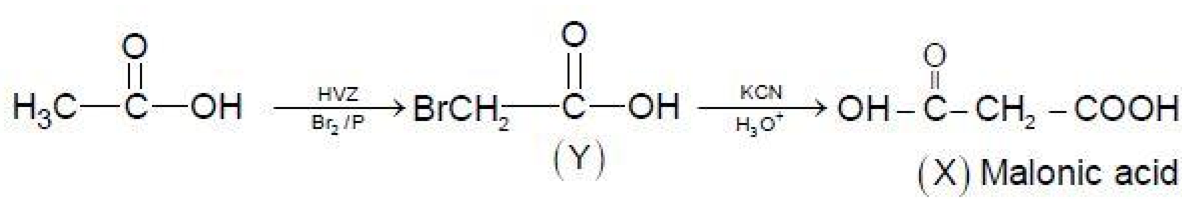
Here (X) is:

(A) Glycollic acid (B) α-hydroxypropionic acid
(C) succinic acid (D) malonic acid

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** D

Solution :-



---

40).

In order to distinguish between $C_2H_5NH_2$ and $C_6H_5NH_2$, which of the following reagents is useful

(A) Heinsberg reagent
(B) p-napthol
(C) Benzene diazonium chloride
(D) None of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

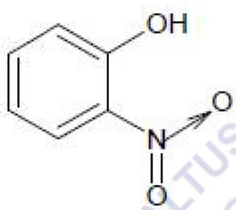
Solution :-

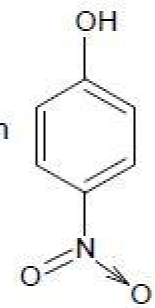
1° aromatic amine on diazotisation followed by coupling with β-napthol gives azo dye test.

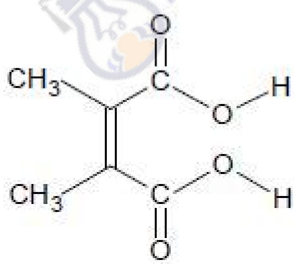
---

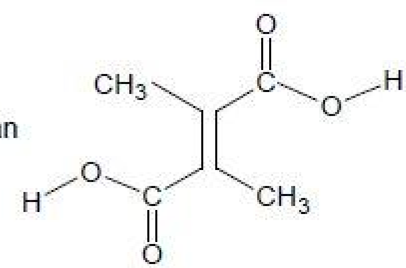
41).

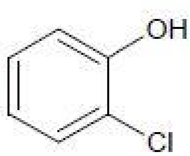
Which of the following statement is false ?

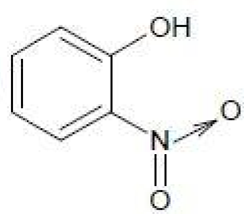
(A) 

has lower boiling point than


(B) 

is less stable than


(C) 

has greater pKa than


(D) H_3BO_3 in C_2H_5OH exists in the cage form due to intermolecular hydrogen bonding.

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** D

Solution :-

(A) Viscosity α intermolecular attraction
α intermolecualr H-bonding tendency

(B) No intramoecular H-bonding in the cis-isomer due to loss in planarity as a result of steric and electronic repulsion.
$\therefore$ stability α symmetry.

(C) Even though o-nitrophenol shows intramolecular H-bonding but has greater – R and – I effect than –Cl.

(D) $H_3BO_3 + 3\ C_2H_5OH \rightarrow B(OC_2H_5)_3 + 3H_2O$.
$\therefore$ No cage lattice structure of H_3BO_3.

---

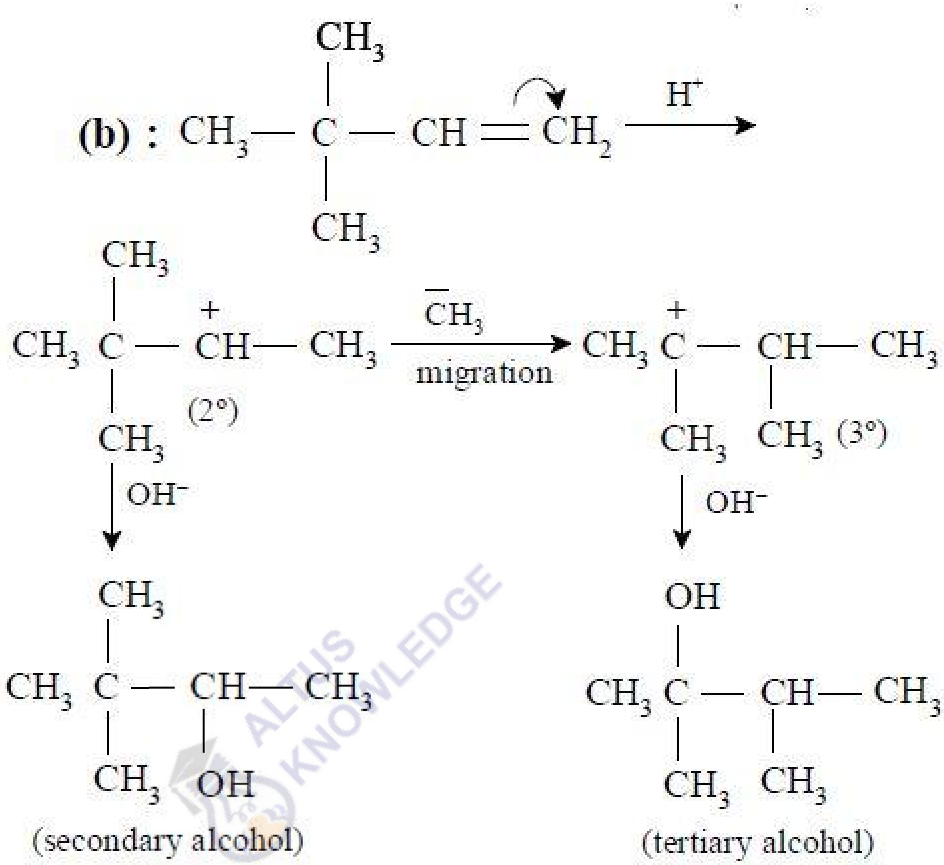
42). Acid catalyzed hydration of alkenes except ethene leads to the formation of
(a) primary alcohol
(b) secondary or tertiary alcohol
(c) mixture of primary and secondary alcohols
(d) mixture of secondary and tertiary alcohols

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

(b) :



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43). Which of the following reactions will yield 2,2-dibromopropane?

(a) $CH_3 - CH = CH_2 \; + \; HBr \rightarrow$

(b) $CH_3 - C \equiv CH \; + \; 2HBr \rightarrow$

(c) $CH_3CH = CHBr \; + HBr \rightarrow$

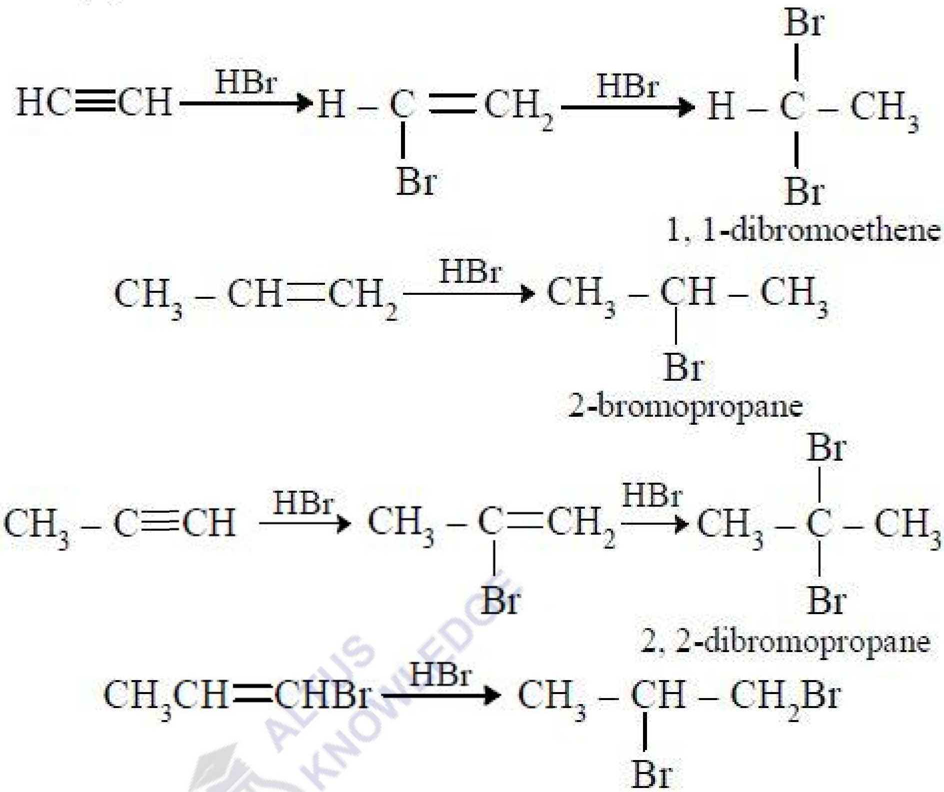
(d) $CH \equiv CH \; + \; 2HBr \rightarrow$

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

(b) :



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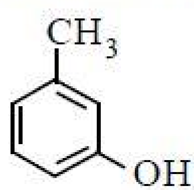
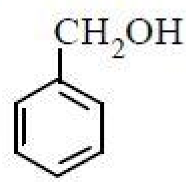
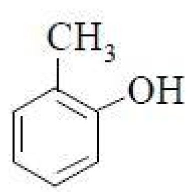
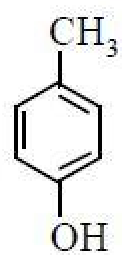
44). The structure of the compound that gives a tribromoderivative on treatment with bromine water is

(a) 

(b) 

(c) 

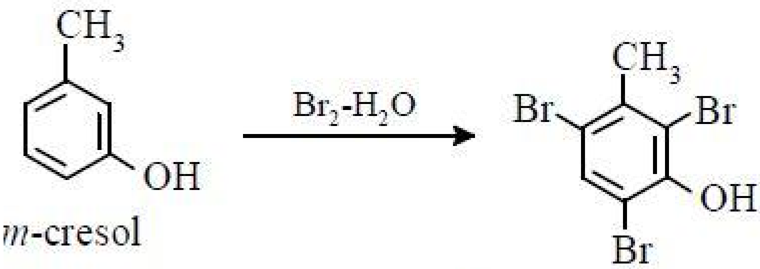
(d) 

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

(a) : Since the compound on treatment with Br_2-water gives a tribromoderivative, therefore it must be *m*-cresol, because it has two *ortho* and one *para* position free with respect to OH group and hence can give tribromoderivative.



2,4,6-tribromo-3-methylphenol

---

45). Fluorobenzene (C_6H_5F) can be synthesised in the laboratory

(a) by heating phenol with HF and KF

(b) from aniline by diazotization followed by heating the diazonium salt with HBF_4

(c) by direct fluorination of benzene with F_2 gas

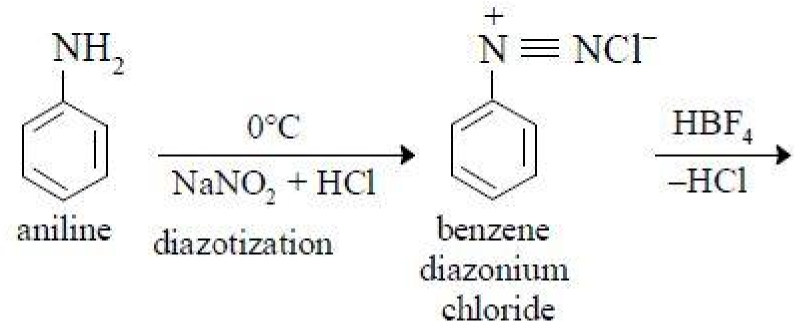
(d) by reacting bromobenzene with NaF solution.

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

(b) :



$$C_6H_5\overset{+}{N}_2BF_4^- \xrightarrow{\Delta} C_6H_5F + BF_3 + N_2$$

benzene diazonium tetra fluoroborate → fluorobenzene

---

46).

In low temperature range fraction of heat supplied to an ideal diatomic gas system, at constant pressure, which bring change in its internal energy is approx.......?

Q.Type:- Digit/Numeric, **Ans:-** 0.71

Solution :-

Heat supplied at constant pressure in the range of temperature, ΔT is ΔH and change in internal energy is ΔU hence $\frac{\Delta U}{\Delta H} = \frac{1}{\gamma} = \frac{1}{1.4} = 0.71(\text{approx})$

---

47).

In a measurement of quantum efficiency of photo-synthesis in green plants, it was found that 10 quanta of red light of wavelength 6850 Å were needed to release one molecule of O_2. The average energy storage in this process is 112 kcal/mol O_2 evolved. What is the energy conversion efficiency in this experiment? Given 1 cal = 4.18 J, $N_A = 6 \times 10^{23}$, $h = 6.64 \times 10^{-34}$ JS.

Q.Type:- Digit/Numeric, **Ans:-** 26.9

Solution :-

$E = \frac{hc}{\lambda} = 2.9 \times 10^{-19}$ J

Total energy of 10 quanta = 10 × 2.9 × 10^{-19}J

Energy stored for process $= \frac{112 \times 4.18 \times 10^3}{6 \times 10^{23}} = 7.8 \times 10^{-19}$ J

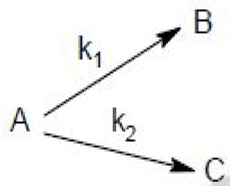
% efficiency $= \frac{7.8 \times 10^{-19}}{29 \times 10^{-19}} \times 100$

= 26.9%

---

48).

For first order parallel reaction k_1 and k_2 are 4 and 2 min^{-1} respectively at 300 K. If the activation energies for the formation of B and C are respectively 30,000 and 33,314 J/mol respectively. The Temperature at which B and C will be obtained in equimolar ratio is:



Q.Type:- Digit/Numeric, **Ans:-** 329.77

Solution :-

$\ell n \frac{k_1'}{k_1} = \frac{E_1}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$ ... (i)

$\ell n \frac{k_2'}{k_2} = \frac{E_2}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$ ... (ii)

Eqn. (ii) – Eqn.(i)

$\ell n \frac{k_2' \times k_1}{k_2 \times k_1'} = \frac{(E_2 - E_1)}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

(For equimolar formation of B and C, $k_2' = k_1'$)

$\ell n\left(\frac{k_1}{k_2}\right) = \left(\frac{8314}{8.314}\right)\left(\frac{T_2 - 300}{300 \times T_2}\right)$

$\ell n 2 = = \left(\frac{8314}{8.314}\right)\left(\frac{T_2 - 300}{300 \times T_2}\right)$

T_2 = 329.77 K.

---

49).

One mole of SO_3 was placed in a two litre vessel at a certain temperature. The following equilibrium was established in the vessel.

$$2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$$

The equilibrium mixture reacted with 0.2 mole $KMnO_4$ in acidic medium. Hence K_c is:

Q.Type:- Digit/Numeric, **Ans:-** 0.125

Solution :-

$$2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$$

at equilibrium $(1-2x)$ $(2x)$ x

Only SO_2 will oxidized.

Equivalent of SO_2 = Equivalent of $KMnO_4$

$2x \times 2 = 0.2 \times 5$

$2x = 0.5$

$$K_c = \frac{\left[\frac{0.5}{2}\right]^2 \left[\frac{0.25}{2}\right]}{\left[\frac{0.5}{2}\right]^2} = 0.125$$

---

50).

The vapour pressure of pure water at 26ºC is 25.21 torr. What is the vapour pressure of a solution which contains 20.0 glucose, $C_6H_{12}O_6$, in 70 g water?

Q.Type:- Digit/Numeric, **Ans:-** 24.5

Solution :-

$$x_{Glocose} = \frac{\left(\frac{20}{180}\right)}{\left(\frac{20}{180}\right)+\left(\frac{70}{18}\right)} = \frac{\left(\frac{1}{9}\right)}{\frac{1}{9}+\frac{70}{18}}$$

$$x_{Glucose} = \frac{\left(\frac{1}{9}\right)}{4} = \frac{1}{36}$$

$$x_{Glucose} = \frac{25.21-P}{25.21} = \frac{1}{36}$$

$$\Rightarrow 25.21 = 25.21 \times 36 - P \times 36$$

$$\Rightarrow P = 24.5$$

Mathematics FL

51).

Consider a circle, $x^2 + y^2 = 1$ and point $P(1, \sqrt{3})$ ·PAB is a secant drawn from P intersecting circle in A and B (distinct) then range of |PA| + |PB| is

(A) $[2, 2\sqrt{3}]$ (B) $(2\sqrt{3}, 4]$

(C) (0, 4] (D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

$$\frac{|PA|+|PB|}{2} > [|PA|\cdot|PB|]^{1/2}$$

$|PA| + |PB| > 2|PT| = 2\sqrt{3}$

Maximum length occurs when PAB passes through centre

i.e. |PA| + |PB| = 4 (Maximum)

So, range is $(2\sqrt{3}, 4]$

---

52).

The total number of 1 word, 2 word, 3 word sentences that can be formed using the letters of the word SAMSUNG is

(A) 8!
(B) 18 × 7!
(C) 11 × 7!
(D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

Required number of sentences = $\left({}^{6}C_0 + {}^{6}C_1 + {}^{6}C_2\right) \times \frac{7!}{2!}$

---

53).

Consider a line $z(i - 1) + \bar{z}(i + 1) = 0$ in the argand plane and a point $z_1 = 2 + 3i$ then the reflection of z_1 in the given line is

(A) 2 – 3i
(B) –2 + 3i
(C) 3 + 2i
(D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

The given line can be written as y = x

Hence reflection is 3 + 2i

---

54).

Let, $t_r = r!$ and $S_n = \sum_{r=1}^{n} r!$ then $\frac{S_n}{24} = a + \frac{\lambda}{24}$; $a, \lambda \in N$ where λ is

(A) 7
(B) 23
(C) 9
(D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

4! = 24

5! = 24 × 5

6! = 24 × 5 × 6

1! + 2! + 3! = 9

So, 1! + 2! + 3! + ….. 100! = 9 + 24p

$\therefore \lambda = 9$

---

55).

The area between the curve $y^2(a + x) = (a - x)^3$ and its vertical asymptote is

(A) $\frac{\pi}{2}a^2$ (B) $2\pi a^2$

(C) $3\pi a^2$ (D) none of these

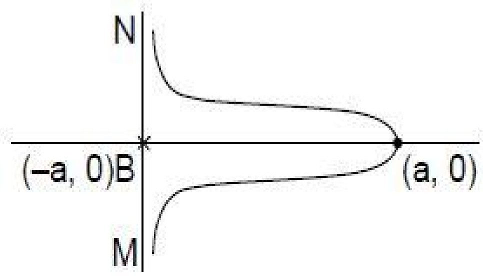
(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

x = –a is the only asymptote to the given curve

$$A = 2\int_{-a}^{a} y\,dx = 3\pi a^2$$



56).

If the tangent to the curve $y = 1 - x^2$ at $x = \alpha (0 < \alpha < 1)$ meets the axes at P and Q. Also α varies, the minimum value of the area of the triangle OPQ is k times the area bounded by the axes and the part of the curve for which $0 < x < 1$, then k is

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{2}{\sqrt{3}}$

(C) $\frac{1}{2}$ (D) $\frac{3}{2}$

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

Area under curve and axis = $\int_0^1 (1 - x^2)dx = \frac{2}{3}$ at point $(\alpha, 1 - \alpha^2)$ tangent is

$[y - (1 - \alpha^2)] = -2\alpha(x - \alpha)$

$\Rightarrow 2\alpha x + y = \alpha^2 + 1 \Rightarrow P \equiv \left(\frac{\alpha^2 + 1}{2\alpha}, 0\right), Q \equiv (0, \alpha^2 + 1)$

Hence, $Q \equiv \left(2, 1 + 4x_1 - x_1^2\right)$

$\therefore A = \left[\left(1 + 2x_1 - x_1^2\right) + \left(1 + 4x_1 - x_1^2\right)\right] = 1 - 3x_1 - x_1^2$

$\frac{dA}{dx_1} = -3 - 2x_1 \Rightarrow x_1 = -\frac{3}{2}$

57).

Consider a parabola $y^2 = \alpha x$ and a point $\left(-\frac{\alpha}{4}, 0\right)$ then midpoint of centres of the circles touching the tangents from given point and its chord of contact is

(A) $\frac{\alpha}{2}$

(B) $\frac{\alpha}{4}$

(C) $\frac{3\alpha}{2}$

(D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** D

Solution :-

$\left(-\frac{\alpha}{4}, 0\right)$ lies on directrix and centre be $(x_1, 0)$. Equation of one of the tangent is $x + y + \frac{\alpha}{4} = 0$ and chord is $x - \frac{\alpha}{4} = 0$, centre is $(x_1, 0)$

$$\frac{\left|x_1 + \frac{\alpha}{4}\right|}{\sqrt{2}} = \frac{\left|x_1 - \frac{\alpha}{4}\right|}{1} \Rightarrow x_1^2 + \frac{\alpha^2}{16} + \frac{\alpha x}{2} = 2x_1^2 + \frac{\alpha^2}{8} - \alpha x$$

$$x_1^2 - \frac{3\alpha x}{2} + \frac{\alpha^2}{16} = 0 \Rightarrow x_1 + x_2 = \frac{3\alpha}{2} \Rightarrow \text{Midpoint} \equiv \left(\frac{3\alpha}{4}, 0\right)$$

58).

Consider the curves C_1: $x^2 + y^2 = 1$ and $C_2 : \frac{x^2}{\sin^2\theta} + \frac{y^2}{\cos^2\theta} = 1$. If a common tangent $y = mx + c$ is drawn to C_1, C_2 then $\left(\frac{\pi}{4} < \theta < \frac{\pi}{2}\right)$

(A) $m \in \phi$

(B) $c = 1$

(C) $m = \frac{1}{\sqrt{2}}$

(D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

For the common tangent $m^2 \sin^2\theta + \cos^2\theta = m^2 + 1$

$m^2(\sin^2\theta - 1) = 1 - \cos^2\theta$

$$\Rightarrow m^2 = \frac{1-\cos^2\theta}{\sin^2\theta - 1} = -\frac{\cos^2\theta}{\sin^2\theta} = -\cot^2\theta$$

Clearly no such m is possible (when $\left(\frac{\pi}{4} < \theta < \frac{\pi}{2}\right)$)

---

59).

Consider a parabola $x^2 = 4y$ and a hyperbola $xy = 1$. A tangent is drawn to parabola meets the hyperbola in A and B then locus of midpoint of AB is

(A) straight line (B) parabola

(C) ellipse (D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

Let midpoint is (h, k) then chord is $y = -\frac{h}{k}x + 2h$ and if it is a tangent then $x^2 = 4\left(-\frac{h}{k}x + 2h\right)$

D = 0

$$\Rightarrow k^2 = -\frac{h}{2} \text{ (Parabola)}$$

---

60).

If $f'\left(x^2 - 4x + 3\right) > 0$, $\forall x \in (2, 3)$; then f(sin x) is increasing on

(A) $\bigcup_{n \in I}\left(2n\pi, (4n+1)\frac{\pi}{2}\right)$ (B) $\bigcup_{n \in I}\left((4n-1)\frac{\pi}{2}, 2n\pi\right)$

(C) R (D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

$x \in (2, 3) \Rightarrow -1 < x^2 - 4x + 3 < 0$, so f(x) is increasing in (–1, 0)

$$\Rightarrow f(\sin x) \text{ is increasing on } \bigcup_{n \in I}\left((4n-1)\frac{\pi}{2}, 2n\pi\right)$$

---

61).

Coordinates of the point on the straight line x + y = 4, which is nearest to the parabola $y^2 = 4(x - 10)$ is

(A) $\left(\frac{17}{2}, -\frac{9}{2}\right)$ (B) (2, 2)

(C) $\left(\frac{3}{2}, \frac{5}{2}\right)$ (D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

Let point P on the straight line x + y = 4 be (m, 4 – m), this will be nearest to the parabola if ⊥ at this point to the straight line becomes normal to the parabola.

Let it is normal at $x - 10 = t^2$, $y = 2t$

Perpendicular to x + y = 4 at (m, 4 – m) is $y - (4 - m) = (x - m)$ .....(1)

Normal at parabola at $(t^2 + 10, 2t)$ is $y + t(x - 10) = 12t + t^3$ .....(2)

(1) and (2) are same $\Rightarrow t = -1$, $m = \frac{17}{2}$ so required point is $\left(\frac{17}{2}, -\frac{9}{2}\right)$

62).

If $\bar{a}$, $\bar{b}$, $\bar{c}$ be three vectors of magnitude $\sqrt{3}$, 1, 2 such that $\bar{a} \times (\bar{a} \times \bar{c}) + 3\bar{b} = 0$, if θ is the angle between $\bar{a}$ and $\bar{c}$, then $\cos^2\theta$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

$$\left|\bar{a} \times (\bar{a} \times \bar{c})\right| = \left|3\bar{b}\right| = 3\left|\bar{b}\right|$$

$$|\bar{a}|.\left|(\bar{a} \times \bar{c})\right| \sin\frac{\pi}{2} = 3.1 \Rightarrow 3 = |\bar{a}|.\left(|\bar{a}| \ |\bar{c}| \sin\theta\right)$$

$$\Rightarrow 3 = 3.2 \sin\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \cos^2\theta = \frac{3}{4}$$

63).

If the function $f : [2, \infty) \rightarrow [1, \infty)$ is defined by $f(x) = 3^{x(x-2)}$, then $f^{-1}(x)$ is

(A) $1 + \sqrt{1+\log_3 x}$ (B) $1 - \sqrt{1+\log_3 x}$

(C) $1 + \sqrt{1-\log_3 x}$ (D) does not exist

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

Let g(x) be the inverse of f, then f(g(x)) = x

$\Rightarrow 3^{g(x)(g(x)-2)} = x$

$\Rightarrow (g(x))^2 - 2g(x) - \log_3 x = 0$

$\Rightarrow g(x) = \dfrac{2 \pm \sqrt{4 + 4\log_3 x}}{2} = 1 \pm \sqrt{1+\log_3 x}$

Since $g : [1, \infty] \rightarrow [2, \infty]$

So $g(x) = 1 + \sqrt{1+\log_3 x}$

---

64).

$\dfrac{1}{x} = \dfrac{2\ \ e}{3\ \ !} + \dfrac{4\ \ e}{5\ \ !} + \dfrac{6\ \ e}{7\ \ !} + \ldots \infty$, then find $\int_0^x f(y)\log_y x\ dy,\ y > 1$

(A) $\dfrac{[f(e)]^2}{2}$ (B) $\dfrac{\left[f\left(\frac{1}{e}\right)\right]^2}{2}$

(C) $\dfrac{\left[f(e^2)\right]^2}{2}$ (D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** D

Solution :-

$$\frac{1}{x} = 2e\left[\frac{1}{3!}+\frac{2}{5!}+\frac{3}{7!}+\ldots\infty\right] = 2eS$$

$$S = \sum_{r=1}^{\infty}\frac{r}{(2r+1)!} = \frac{1}{2}\sum_{r=1}^{\infty}\frac{(2r+1)-1}{(2r+1)!} = \frac{1}{2}\sum_{r=1}^{\infty}\left[\frac{1}{(2r)!}-\frac{1}{(2r+1)!}\right]$$

$$= \frac{1}{2}\left[\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\ldots\infty\right]$$

$$= \frac{1}{2}e^{-1} = \frac{1}{2e}$$

$$\Rightarrow \frac{1}{x} = 2eS = \frac{2e}{2e} = 1$$

$$\text{So, } \int_0^1 f(y)\log_y x\,dy = 0$$

---

65).

If lines $x = y = z$, $x = \frac{y}{2} = \frac{z}{3}$ and third line passing through (1, 1, 1) form a triangle of area $\sqrt{6}$ units then point of intersection of third line with second line will be

(A) (1, 2, 3)
(B) (2, 4, 6)
(C) $\left(\frac{4}{3}, \frac{8}{3}, \frac{12}{3}\right)$
(D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

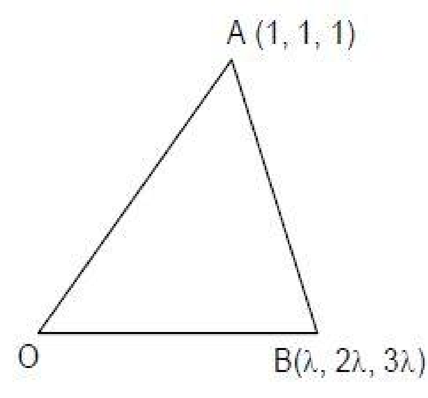
Let any point on second line be $(\lambda, 2\lambda, 3\lambda)$

$\cos\theta = \frac{6}{\sqrt{42}}$ $\sin\theta = \frac{6}{\sqrt{42}}$

$\Delta_{OAB} = \frac{1}{2}(OA).OB\sin\theta = \frac{1}{2}\sqrt{3}.\lambda\sqrt{14}\times\frac{6}{\sqrt{42}} = \sqrt{6}$

$\Rightarrow \lambda = 2$

So, B is (2, 4, 6)



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66).

Let f(x) = max{tan x, cot x}. Then number of roots of the equation $f(x) = \frac{1}{\sqrt{3}}$ in $(0, 2\pi)$ is

(A) 2 (B) 4
(C) 0 (D) infinite

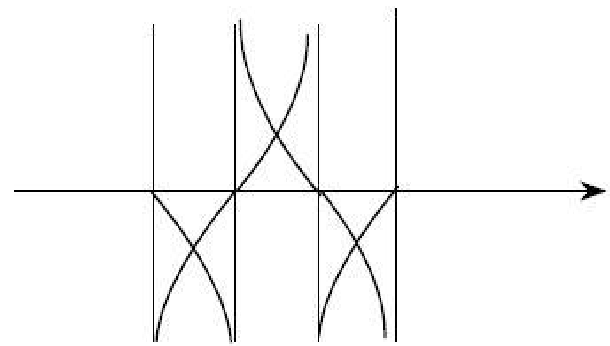
(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

If we draw the graph of tan x and cot x, we observes that range of f (x) is $[-1, 0) \cup [1, \infty)$

So $f(x) = \frac{1}{\sqrt{3}}$ does not have any root.



67).

If $f(x) = \int_0^4 e^{|t-x|}dt$ $(0 \le x \le 4)$, the maximum value of f(x) is

(A) $e^4 - 1$ (B) $2(e^2 - 1)$
(C) $e^2 - 1$ (D) none of these

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

$$f(x) = \int_0^x e^{|t-x|}dt + \int_x^4 e^{|t-x|}dt = \int_0^x e^{x-t}dt + \int_x^4 e^{t-x}dt = -e^{x-t}\Big|_0^x + e^{t-x}\Big|_x^4 = e^x + e^{4-x} - 2$$

$f'(x) = e^x - e^{4-x} = 0 \Rightarrow x = 4 - x \Rightarrow x = 2$

$f(0) = f(4) = e^4 - 1$, $f(2) = 2(e^2 - 1)$, so maximum value of f(x) is $e^4 - 1$.

68).

The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$

(a) $\frac{1}{4}e^{-2x}$ (b) $\frac{1}{4}e^{-2x} + cx + d$

(c) $\frac{1}{4}e^{-2x} + cx^2 + d$ (d) $\frac{1}{4}e^{-2x} + c + d.$

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** B

Solution :-

(b) : Given $\frac{d^2y}{dx^2} = e^{-2x}$

$\therefore \quad \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c$

$\therefore \quad y = \frac{e^{-2x}}{4} + cx + d$

---

69). The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is

(a) $4 \le x^2 + y^2 \le 64$ (b) $x^2 + y^2 \le 25$

(c) $x^2 + y^2 \ge 25$ (d) $3 \le x^2 + y^2 \le 9$

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** A

Solution :-

(a) : Let (α, β) is the centre of the circle whose radius is 3.

$\therefore$ Equation of such circle be

$(x - \alpha)^2 + (y - \beta)^2 = 3^2$

$\Rightarrow \quad \alpha^2 + \beta^2 - 2\alpha x - 2\beta y + 25 = 9$

$\Rightarrow \quad x^2 + y^2 - 2x^2 - 2y^2 + 25 = 9$

$\Rightarrow \quad x^2 + y^2 = 25 - 9$

$\Rightarrow \quad x^2 + y^2 = 16$ and $x^2 + y^2 = 25$

$\Rightarrow \quad 4 \le x^2 + y^2 \le 64$

---

70). The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d$ will be perpendicular, if and only if

(a) $aa' + bb' + cc' = 0$
(b) $(a + a')(b + b') + (c + c') = 0$
(c) $aa' + cc' + 1 = 0$
(d) $aa' + bb' + cc' + 1 = 0$.

(a) A **(b)** B **(c)** C **(d)** D

Q.Type:- MCQ Single, **Ans:-** C

Solution :-

(c) : Given lines can be written as

$$\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c} \text{ and}$$

$$\frac{x-b'}{a'} = \frac{y-0}{1} = \frac{z-d}{c'}$$

$\therefore$ Required condition of perpendicularity is

$aa' + cc' + 1 = 0$

---

71).

Consider a curve $|z - i| = 2$ and a point $z_1 = 3 - i$, then the length of tangent made from the point (z_1) to the curve is

Q.Type:- Digit/Numeric, **Ans:-** 3

Solution :-

Length of tangent is = $\sqrt{|z_1 - i|^2 - 4}$

$= \sqrt{|3 - 2i|^2 - 4} = \sqrt{9 + 4 - 4} = 3$

---

72).

The sequence a_n is defined by $a_1 = \frac{1}{2}$, $a_{n+1} = a_n^2 + a_n$. Also, $S = \frac{1}{a_1 + 1} + \frac{1}{a_2 + 1} + ... + \frac{1}{a_{100} + 1}$

then [S] (where [.] denotes the greatest integer function) is

Q.Type:- Digit/Numeric, **Ans:-** 1

Solution :-

$$\frac{1}{a_{n+1}} = \frac{1}{a_n(a_n+1)} = \frac{1}{a_n} - \frac{1}{a_n+1}$$

$$\Rightarrow S = \frac{1}{a_1} - \frac{1}{a_2} + ... + \frac{1}{a_{100}} - \frac{1}{a_{101}}$$

$$= \frac{1}{a_1} - \frac{1}{a_{101}} = 2 - \frac{1}{a_{101}}$$

Since, $a_{101} > 1 \Rightarrow [S] = 1$

---

73). If $\Delta(x) = \begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \ln(1+x) & \cos x & \sin x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix} = A + Bx + Cx^2 + \ldots\ldots.,$ then B is equal to

Q.Type:- Digit/Numeric, **Ans:-** 0

Solution :-

$$\Delta'(x) = \begin{vmatrix} e^x & 2\cos 2x & 2x\sec^2 x^2 \\ \ln(1+x) & \cos x & \sin x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix} + \begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \frac{1}{(1+x)} & -\sin x & \cos x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix} + \begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \ln(1+x) & \cos x & \sin x \\ -2x\sin x^2 & e^x & 2x\cos x^2 \end{vmatrix}$$

$= B + 2Cx + ...$

Put $x = 0$, $B = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 0$

---

74). $\int_{\pi}^{10\pi} |\sin x|\, dx$ is

Q.Type:- Digit/Numeric, **Ans:-** 18

Solution :-

$$\int_{\pi}^{10\pi} |\sin x|\, dx$$

$$= \int_{0}^{10\pi} |\sin x|\, dx - \int_{0}^{\pi} |\sin x|\, dx$$

$$= 10 \times 2 - 1 \times 2$$

$$= 18 \qquad \text{(Using period of } |\sin x| = \pi\text{)}$$

---

75). If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, \vec{a}, \vec{a}^2)$, $(1, \vec{b}, \vec{b}^2)$ and $(1, \vec{c}, \vec{c}^2)$ are non-coplanar, then the product abc equals

Q.Type:- Digit/Numeric, **Ans:-** -1

Solution :-

As vectors $(1, \vec{a}, \vec{a}^2)$, $(1, \vec{b}, \vec{b}^2)$, $(1, \vec{c}, \vec{c}^2)$ are non coplanar.

$$\therefore \quad \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \qquad \ldots (A)$$

$$\text{now} \quad \begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = 0$$

On solving, we get

$$\Rightarrow (1 + abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$\Rightarrow (1 + abc) = 0$ by using (A)